Thermodynamic properties and phase transition of strongly interacting two-component Fermi gases

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The thermodynamic properties of two-component Fermi gases with divergent scattering length is investigated and the transition temperature for the emergence of a stable dimeric gas is obtained by a simple theoretical model where the unique property of the dimers (*i.e.* fermionic atom pairs) with divergent scattering length is considered. Below the transition temperature, through the investigation of the overall entropy for the mixture gas of fermionic atoms and dimers, we calculate the relation between the energy and temperature of the system. In the limit of zero temperature, based on the chemical equilibrium condition, the fraction of the dimers is investigated and the role of the dimers in the collective excitations of the system is also discussed. It is found that our theoretical results agree with the condensate fraction, collective excitations, transition temperature and heat capacity investigated by the recent experiments about the strongly interacting two-component Fermi gases.

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I. INTRODUCTION

It is a quite challenging theoretical problem to explain the basic properties of the strongly interacting many-body system because there is no exact solution for this nonlinear many-body system and the ordinary perturbation method can not give us a reliable quantitative theoretical predication. The strongly interacting Fermi gases is a very important theoretical problem because the atomic nucleus, quark-gluon plasma and high temperature superconductivity *etc* relate closely to the strongly interacting Fermi gases. Due to the strong interaction, a phenomenological theory would contribute largely to our understanding of this complex system. For example, for the atomic nucleus which is a complex many-body system bound by the strong interaction, there is a successful phenomenological theory by introducing an effective force. Obviously, clear experimental investigations can give us important clues to find a reasonable phenomenological model and even a theory to reveal the microscopic mechanism accounting for the basic phenomena of the strongly interacting many-body system. In the last few years, the experimental advances on the ultracold strongly interacting two-component Fermi gases such as [1] provide us an important opportunity to develop a theory of the strongly interacting system.

There is a broad interest in the ultracold two-component Fermi gases because the two-component Fermi gases can be cooled to a temperature far below the Fermi temperature so that there is an obvious quantum statistical effect. Another special interest in this system lies in that through a magnetic-field Feshbach resonance, one can change in a controllable way the value and even sign of the scattering length a between atoms. For two-component Fermi gases, when the magnetic field is tuned so that the energy of a quasibound molecular state in a closed channel matches the total energy in an open channel, there is a magnetic-field Feshbach resonance. Below the Feshbach resonant magnetic field (BEC side), the interaction between atoms is repulsive and there exists molecule which is a short-range fermionic atom pairs. Below the critical temperature, there is a molecular Bose-Einstein condensation (BEC) which has been observed in the recent experiments [2, 3, 4]. Above the Feshbach resonant magnetic field (BCS side), the interaction between atoms is attractive and there would be a Bardeen-Cooper-Schrieffer (BCS) superfluid behavior due to the atomic Cooper pairs at sufficient low temperature. Recently, by tuning the uniform magnetic field near the resonant magnetic field, there are a lot of interesting experimental works on the BCS-BEC crossover [5, 6, 7, 8, 9, 10, 11, 12] such as the measurement of the cloud size [5], condensate fraction [6, 7], collective excitation [8, 9], pairing gap [10], and heat capacity [12] *etc.* Besides the pioneering theoretical works [13, 14, 15], in the last few years, there are intensive theoretical investigations on the BCS-BEC crossover [16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35] such as resonance superfluid [17, 18] and universal behavior for the gases with divergent scattering length [28, 29, 30, 31, 32].

In the BCS-BEC crossover regime, the scattering

length becomes divergent on magnetic-field Feshbach resonance, thus the system on resonance gives us an ideal system to investigate directly the strongly interacting many-body system. In the limit of this strong interaction, the scattering length becomes divergent and thus it should not appear in the final expression of a physical quantity. For the ultracold gases with divergent scattering length, it is a quite attractive theoretical problem considering the possibility that a phenomenological theoretical model would be very simple because the scattering length will not appear in the final expression of the physical quantity although it is the most important parameter to show the interaction between particles. In the present work, we investigate theoretically the thermodynamic properties of the two-component Fermi gases in the case of the scattering length being much larger than the average distance $\overline{l}$ between particles. On the side of weakly attractive (or repulsive) interaction, there are pairs of atoms in the form of atomic Cooper pairs (or molecules) which has been verified through the experimental investigation of the condensation fraction [6, 7] and pairing gap [10]. Generalizing this observation, we assume here that there are dimers which comprise two fermionic atoms with different spin state for the system on magnetic-field Feshbach resonance. Most recently, there is a measurement of the heat capacity [12] for strongly interacting Fermi gases of ^6Li atoms which gives clearly the evidence of a phase transition for the emergence of the fermion pairs. In the present work, at finite temperature as well as zero temperature, the thermodynamic properties of the system such as the entropy, transition temperature, fraction of dimers are investigated based on a simple theoretical model that the system is in a mixture of the Fermi gases and dimeric gas below the transition temperature.

In Sec. II, at zero temperature, we consider the thermodynamic properties for the mixture gases of fermionic atoms and dimers based on the chemical equilibrium condition. It is found that this theoretical model is consistent with the recent experimental investigation of the condensate fraction [7] and collective excitations [8, 9]. The coexistence of Fermi gases and dimeric gas at zero temperature means that there should be a phase transition for the emergence of dimeric gas at finite temperature. In Sec. III, when the thermal excitation of the dimers is considered, we give a transition temperature that below this transition temperature there would be a stable dimeric gas. The fraction of dimers below the transition temperature is also given. In Sec. IV, through the investigation of the overall entropy of the Fermi gases and dimeric gas, we calculate the relation between the energy and temperature of the system. The role of the dimeric gas is discussed and found that the theoretical result based on the presence of the dimeric gas is in agreement with the recent experiment about heat capacity [12]. Finally, we give a brief summary and discussion in Sev. V.

II. MIXTURE OF TWO-COMPONENT FERMI GASES AND DIMERIC GAS AT ZERO TEMPERATURE

In the experiment [12] about the heat capacity, the phase transition for the strongly interacting Fermi gases is observed clearly at a transition temperature. If there is a phase transition for the emergence of a stable dimeric gas at a transition temperature, there would be a mixture of two-component Fermi gases and dimeric gas below the transition temperature, and the evidence for the mixture of two-component Fermi gases and dimeric gas would give us an important evidence that there is a phase transition for the emergence of the stable dimeric gas. To give a clear presentation, we first propose in this section a theoretical model based on the mixture of two-component Fermi gases and dimeric gas at zero temperature [32], while in the following sections we will consider the thermodynamic properties of this mixture gas at finite temperature.

In the present experiments on two-component Fermi gases, an equal mixture of Fermi gases is prepared below the Fermi temperature. Assuming that N is the total number of fermionic atoms without dimers, while N_F and N_D are the number of fermionic atoms and dimers, we have

$$N = N_F + 2N_D = 2N_{F\uparrow} + 2N_D, \tag{1}$$

where $N_{F\uparrow}$ $(= N_{F\downarrow})$ being the number of fermionic atoms in a spin state, while $N_F = 2N_{F\uparrow}$ being the total number of fermionic atoms.

At zero temperature, for the mixture of two-component Fermi gases and dimeric gas, the dynamic equilibrium is characterized by the fact that the Gibbs free energy of the system is a minimum. Assuming that $\mu_{F\uparrow}$ and $\mu_{F\downarrow}$ being the chemical potential of the Fermi gases and μ_D being the chemical potential of the dimeric gas, the minimum of the Gibbs free energy means that

$$2\mu_{F\uparrow} = 2\mu_{F\downarrow} = \mu_D. \tag{2}$$

The chemical potential of the dimeric gas is $\mu_D = \varepsilon_D + \mu_t$ with ε_D being the dimeric binding energy and μ_t being the contribution to the chemical potential due to the thermal equilibrium of the dimeric gas.

For the unitarity limit that the absolute value $|a|$ of the scattering length is much larger than the average distance $\overline{l}$ between atoms, although the scattering length a will play an important role, it will not appear in the final result of a physical quantity such as the chemical potential because it can be regarded as infinity. In this case, the length scale $\overline{l} \sim n_{F\uparrow}^{-1/3}$ ($n_{F\uparrow}$ is the density distribution of the fermionic atoms in a spin state) rather than a will appear in the final result of a physical quantity which means a universal behavior for a system. One can get a rough expression for the

chemical potential through a dimensional analysis that $\mu_{F\uparrow} \sim (\Delta p)^2/2m \sim \hbar^2/2m\bar{l}^2 \sim \hbar^2 n_{F\uparrow}^{2/3}/2m$. Based on this dimensional analysis and local density approximation, at zero temperature, one can get the following form of the chemical potential $\mu_{F\uparrow}$ $(= \mu_{F\downarrow})$ for the Fermi gas:

$$\mu_{F\uparrow} = (1+\beta_F)\frac{\hbar^2\left(6\pi^2\right)^{2/3}}{2m}n_{F\uparrow}^{2/3} + V_{ext}(\mathbf{r}), \quad (3)$$

where $V_{ext}(\mathbf{r}) = m\left(\omega_x^2x^2+\omega_y^2y^2+\omega_z^2z^2\right)/2$ is the external potential of the fermionic atoms. Without β_F, the above expression gives the chemical potential of an ideal Fermi gas in the local density approximation. The parameter β_F shows the role of the extremely large scattering length a in the unitarity limit. The parameter β_F was first measured in [1] with a careful experimental investigation of the strongly interacting two-component Fermi gases near the Feshbach resonance. β_F has been also calculated with different theoretical methods [28, 29]. In [29], $\beta_1 = -0.56$ based on a quantum Monte Carlo calculation. From Eq. (3), the chemical potential at zero temperature is given by

$$\mu_{F\uparrow}(T=0) = \mu_{F\downarrow}(T=0) = \sqrt{1+\beta_F}T_Fk_B, \quad (4)$$

where $T_F = (6N_{F\uparrow})^{1/3}\hbar\omega_{ho}/k_B$ with $\omega_{ho} = (\omega_x\omega_y\omega_z)^{1/3}$. In getting the above result, we have used a useful mathematical technique that with the definition of the effective mass $m_F^* = m/(1+\beta_F)$ and effective angular frequency $\omega_x^* = \sqrt{1+\beta_F}\omega_x$, $\omega_y^* = \sqrt{1+\beta_F}\omega_y$, $\omega_z^* = \sqrt{1+\beta_F}\omega_z$ so that $\mu_{F\uparrow}$ in Eq. (3) becomes the expression of an ideal Fermi gas about the effective mass m_F^* and effective angular frequency. This mathematical technique for the definition of the effective mass and effective angular frequency will be used in the whole paper.

On resonance, because the scattering length between fermionic atoms with different spin state is divergent, the scattering length a_D between dimers can be also regarded as divergent. In the unitarity limit, for the dimeric gas, the scattering length a_D should not appear in the final result of a physical quantity too. We assume here that the dimeric gas shows an analogous universal behavior with the Fermi gas. Through the dimensional analysis analogous to the Fermi gas in the unitarity limit, the chemical potential μ_D of the dimeric gas at zero temperature is given by

$$\mu_D = (1+\beta_D)\frac{\hbar^2\left(6\pi^2\right)^{2/3}}{2\times 2m}n_D^{2/3} + 2V_{ext}(\mathbf{r}) + \varepsilon_D, \quad (5)$$

where n_D is the density distribution of the dimeric gas. The parameter β_D in μ_D is also due to the large scattering length a_D between dimers and it can be calculated in the unitarity limit. Near the Feshbach resonance, the dimeric energy $\varepsilon_D \sim \hbar^2/ma^2$. Thus, on resonance, ε_D can be omitted in the above expression. From the above equation, one can get the chemical potential for the dimeric gas at zero temperature:

$$\mu_D(T=0) = \sqrt{1+\beta_D}T_Dk_B, \quad (6)$$

where $T_D = (6N_D)^{1/3}\hbar\omega_{ho}/k_B$ with N_D being the number of dimers. In getting the above result, we have used a useful mathematical technique that the chemical potential becomes the form of an ideal gas about the effective mass $m_D^* = 2m/(1+\beta_D)$ and effective angular frequency $\omega_x^* = \sqrt{1+\beta_F}\omega_x$, $\omega_y^* = \sqrt{1+\beta_F}\omega_y$, $\omega_z^* = \sqrt{1+\beta_F}\omega_z$ of the dimer.

From the dynamic equilibrium condition $2\mu_{F\uparrow} = \mu_D$ and the chemical potential given by Eqs. (4) and (6), one gets the following general equation to determine the fraction of dimers at zero temperature and on Feshbach resonance:

$$4\beta_r(1-x)^{2/3} = x^{2/3}, \quad (7)$$

where $\beta_r = (1+\beta_F)/(1+\beta_D)$ and the fraction of dimers $x = 2N_D/N$.

In the presence of dimers, the chemical potential of the system is obviously lower than the case without dimers. Thus, the mixture gas of the fermionic atoms and dimers is a stable state of the system. For $|a| >> \overline{l}$, the fraction of the dimeric gas is a general result once the system is in the unitarity limit. One can see easily from Eq. (7) that there is a quite high fraction of dimers if the value of β_D is close to β_F. An indirect evidence of the dimeric gas is given by a recent experiment [7] that there is significant fraction of the molecules in zero-momentum state after a fast magnetic field transfer (the magnetic field is swept below the Feshbach resonant magnetic field B_0 so that the scattering length between fermionic atoms becomes positive) to create bound molecules from the dimers. After the dimer-molecule conversion, in [7] the maximum fraction of the molecules in zero-momentum state is observed to be 80%. In this case, β_r is estimated to be 0.63.

At zero temperature and thus the thermal excitation is omitted, the dimer comprising two fermionic atoms is quite stable. Because the dimeric gas is immersed in the degenerate two-component Fermi gases, due to Pauli blocking comes from the degenerate Fermi gases, any dimer can not be dissociated into two fermionic atoms once the equilibrium is attained so that $2\mu_{F\uparrow} = \mu_D$. In fact, the stability of the dimeric gas is consistent with the experiment [7] that extremely close to the Feshbach resonance there is no obvious decreasing of the molecules in zero-momentum state after the dimer-molecule conversion process even after 10 s hold time of the final magnetic field. Due to the stability and Pauli blocking, the dimer-molecule conversion will always convert the fermionic atom pairs in a same dimer into bound molecules. Thus at zero temperature, all the dimers will be converted into molecules with zero-momentum

state during the dimer-molecule conversion process. This means that the fraction of dimers in thermal equilibrium investigated here can be used to explain the experimental result of the fraction of zero-momentum molecules after the dimer-molecule conversion.

In the recent experiments [8, 9], the frequency of a radial breathing mode is observed extremely close to the Feshbach resonance and found to agree well with the theoretical predication based on hydrodynamic theory in the unitarity limit [25]. Combining with the experimental result in [7] that there is a high fraction of molecules in zero-momentum state after the dimer-molecule conversion process, we see that the dimeric gas should play a dominant role in determining the frequency of the collective oscillations due to its high fraction. For the radial breathing mode, the agreement of the experiment with theoretical result in the unitarity limit shows that it is reasonable to describe the dimeric gas by the chemical potential given by Eq. (5). In fact, an analogous form of Eq. (5) is used to calculate the frequency of the radial breathing mode in [25], whose result agrees well with the experiment in [8, 9]. Different from the theoretical model in [25], however, in the present work on resonance, the dimeric gas plays a dominant role in the frequency of the collective oscillations at zero temperature, rather than the Fermi gases.

III. PHASE TRANSITION FOR THE EMERGENCE OF STABLE DIMERIC GAS

On resonance, the divergent scattering length between dimers would make the property of the dimeric gas become quite unique. From the form of the chemical potential (5) for the dimeric gas which is consistent with the experiments about the condensate fraction [7] and collective excitations [8, 9], the dimeric gas is described by a density of states analogous to that of a Fermi gas. This unique property of the dimeric gas is due to the divergent scattering length between dimers, and is enlightened by the well-known result that a one-dimensional strongly interacting Bose gas behave like Fermi gases [36, 37] which has been verified by recent experiments [38, 39]. Based on these considerations, we assume here that the dimers with divergent scattering length would be described by the Fermi statistics although the dimer is a boson. Obviously, the validity of this assumption should be finally tested by experiments.

Based on the above assumption that the dimers satisfy Fermi statistics, at finite temperature, the occupation number of the dimers at a state of the energy level ε_D is given by

$$f_D\left(\varepsilon_D\right)=\frac{1}{e^{\left(\varepsilon_D-\mu_D(T)\right)/k_BT}+1}, \tag{8}$$

while the occupation number of the fermionic atoms at a state of the energy level ε_F is given by

$$f_{F\uparrow}\left(\varepsilon_F\right)=f_{F\downarrow}\left(\varepsilon_F\right)=\frac{1}{e^{\left(\varepsilon_F-\mu_{F\uparrow}(T)\right)/k_BT}+1}. \tag{9}$$

In Eqs. (8) and (9), the strong interaction has been considered through the chemical potential μ_D and $\mu_{F\uparrow}$ which depends respectively on the parameters β_D and β_F.

For the state below the energy level ε_F for the fermionic atoms and ε_D for the dimers, if the occupation number of the fermionic atoms and dimers in a state are both 1, the dimers can not be dissociated into two fermionic atoms below the energy level ε_F because of Pauli exclusion principle. However, it is possible that the dimers can be thermally excited to high energy level and dissociated into two fermionic atoms especially because the binding energy of the dimers can be omitted on resonance. Thus, sufficient low temperature is needed for the existence of a stable dimeric gas. Assuming that ε_F^c and ε_D^c are the critical energy level that below these energy level there is a stable coexistence of two-component Fermi gases and dimeric gas, due to the fact that the dimer comprises two fermionic atoms with different spin state, one has $\varepsilon_D^c=2\varepsilon_F^c$. One can also understand this relation from the chemical equilibrium condition $\mu_D=2\mu_{F\uparrow}$. The stability of the coexistence of these gases in the presence of thermal excitation would give us a strong confinement condition on the value of ε_F^c and ε_D^c. At finite temperature, the thermally excited energy for fermionic atom and dimer is respectively given by $\varepsilon_{therm}^F=3k_BT/2$ and $\varepsilon_{therm}^D=3k_BT$. $\varepsilon_{therm}^D/\varepsilon_{therm}^F=2$ is due to the fact that a dimer comprises two fermionic atoms.

Based on the above analyses, ε_F^c and ε_D^c can be determined through the following equation:

$$2f_{F\uparrow}\left(\varepsilon_F^c+\varepsilon_{therm}^F\right)+2f_D\left(\varepsilon_D^c+\varepsilon_{therm}^D\right)=3. \tag{10}$$

The left hand side of the above equation shows the overall number of atoms (Note that a dimer comprises two fermionic atoms). The factor 3 on the right hand side of the above equation is due to Pauli exclusion principle for the fermionic atoms and the assumption that the dimer satisfies Fermi statistics in this paper. When the confinement condition given by Eq. (10) is satisfied, the dimers below the energy level ε_D^c can not be effectively thermally excited and dissociated into two fermionic atoms.

From Eqs. (8) and (9), Eq. (10) can be given more explicitly by

$$\frac{2}{e^{\left[\varepsilon_F^c+\frac{3}{2}k_BT-\mu_{F\uparrow}(T)\right]/k_BT}+1}+$$

$$\frac{2}{e^{\left[\varepsilon_D^c+3k_BT-\mu_D(T)\right]/k_BT}+1}=3. \tag{11}$$

At finite temperature, the chemical potential $\mu_{F\uparrow}\left(T\right)$ is estimated to be:

$$\mu_{F\uparrow}(T)=\mu_{F\uparrow}(T=0)\left[1-\frac{\pi^2}{3}\left(\frac{k_BT}{\mu_{F\uparrow}(T=0)}\right)^2\right]. \tag{12}$$

The transition temperature for the emergence of a stable dimeric gas is then obtained by setting $\varepsilon_F^c=0$ and $\varepsilon_D^c=0$ in Eq. (11). Using $\mu_D(T)=2\mu_{F\uparrow}(T)$ and assuming that $\xi=k_BT/\mu_{F\uparrow}(T=0)=T/\sqrt{1+\beta_F}T_F$, the critical value ξ_0 is then determined by

$$\frac{2}{e^{\left[3/2-\left(1-\pi^2\xi_0^2/3\right)/\xi_0\right]}+1}+\frac{2}{e^{\left[3-2\left(1-\pi^2\xi_0^2/3\right)/\xi_0\right]}+1}=3. \tag{13}$$

In this case, one find that $\xi_0=0.31$, which agrees very well with the experimental result 0.33 [12]. This theoretical value of ξ_0 also shows that one can use the approximate expression for the chemical potential $\mu_{F\uparrow}(T)$ at finite temperature given by Eq. (12) because for this value of ξ_0 the high-order term of the chemical potential $\mu_{F\uparrow}(T)$ can be omitted.

Below the transition temperature, one can get the critical energy level for the dimers from Eq. (11) which is given by

$$\varepsilon_D^c=\mu_D(T=0)\left[1+\left(\frac{\pi^2\xi_0}{3}-\frac{1}{\xi_0}\right)\frac{2k_BT}{\mu_D(T=0)}\right.$$

$$\left.-\frac{4\pi^2}{3}\left(\frac{k_BT}{\mu_D(T=0)}\right)^2\right]. \tag{14}$$

When the dimers satisfy Fermi statistics, the density state of the dimers is proportional to ε_D^2. The relation between the number of stable dimers and temperature is then given by

$$N_D(T)=N_D(T=0)\left[1+\left(\frac{\pi^2\xi_0}{3}-\frac{1}{\xi_0}\right)\xi-\frac{\pi^2\xi^2}{3}\right]^3, \tag{15}$$

where $N_D(T=0)$ is the number of dimers at zero temperature.

IV. THE OVERALL ENTROPY OF TWO-COMPONENT FERMI GASES AND DIMERIC GAS

The entropy of two-component Fermi gases takes the following form

$$S_F=-2k_B\sum_{\lambda_{F\uparrow}}\left[f_{\lambda_{F\uparrow}}\ln f_{\lambda_{F\uparrow}}+\left(1-f_{\lambda_{F\uparrow}}\right)\ln\left(1-f_{\lambda_{F\uparrow}}\right)\right], \tag{16}$$

where $\lambda_{F\uparrow}$ denotes the state of the fermionic atom and $f_{\lambda_{F\uparrow}}$ is the occupation number of the fermionic atom in the state $\lambda_{F\uparrow}$. The factor 2 is due to two spin states of the fermionic atom. For the dimeric gas, under the assumption that the dimers satisfy the Fermi statistics, the entropy of the dimeric gas is given by

$$S_D=-k_B\sum_{\lambda_D}\left[f_{\lambda_D}\ln f_{\lambda_D}+(1-f_{\lambda_D})\ln(1-f_{\lambda_D})\right], \tag{17}$$

where λ_D denotes the state of the dimers and f_{λ_D} is the occupation number of the dimer in the state λ_D. The overall entropy S_{F-D} of the system is then given by

$$S_{F-D}=S_F+S_D. \tag{18}$$

To show the role of the dimeric gas in the thermodynamic properties of the system, we first consider the entropy of the system for the temperature far below the Fermi temperature T_F (or T_D for the dimeric gas). In this case, to the first order approximation, the overall entropy S_{F-D} is given by

$$S_{F-D}=\frac{\pi^2N_Fk_BT}{\sqrt{1+\beta_F}T_F}+\frac{\pi^2N_Dk_BT}{\sqrt{1+\beta_D}T_D}. \tag{19}$$

In the above equation for the entropy, the second term represents the contribution of the dimeric gas. Using the condition of the chemical equilibrium given by Eq. (2), one has

$$S_{F-D}=\frac{\pi^2(2N_F+N_D)k_BT}{2\sqrt{1+\beta_F}T_F}. \tag{20}$$

Using the formula $\partial S_{F-D}/\partial E=1/T$, we have

$$\frac{\partial S_{F-D}}{\partial E}=\frac{\partial S_{F-D}}{\partial T}\frac{\partial T}{\partial E}=\frac{1}{T}. \tag{21}$$

Thus

$$\frac{\partial E}{\partial T}=T\frac{\partial S_{F-D}}{\partial T}. \tag{22}$$

In the limit of zero temperature, omitting the high order terms of the partial differential $\partial S_{F-D}/\partial T$, one has

$$\frac{\partial S_{F-D}}{\partial T}=\frac{\pi^2(2N_F+N_D)k_B}{2\sqrt{1+\beta_F}T_F}. \tag{23}$$

From the above equation, we get

$$\frac{\partial E}{\partial T}=\frac{\pi^2(2N_F+N_D)k_BT}{2\sqrt{1+\beta_F}T_F}. \tag{24}$$

In the limit of zero temperature, E is then given by:

$$E = E_0 + \frac{\pi^2 \left(2N_F + N_D\right) k_B T^2}{4\sqrt{1+\beta_F} T_F}, \tag{25}$$

where E_0 is the total ground state energy of the mixture gases at zero temperature. After a simple calculation, one gets

$$E_0 = \frac{3N\sqrt{1+\beta_F} k_B T_F}{4}. \tag{26}$$

From Eqs. (25) and (26), we have

$$\frac{E}{E_0} - 1 = \frac{2\pi^2 \alpha}{3} \left(\frac{T}{T_F}\right)_{fit}^2, \tag{27}$$

where $\alpha = 1 - 3N_D\left(T\right)/2N$ and $\left(T/T_F\right)_{fit} \equiv \xi = T/\sqrt{1+\beta_F} T_F$. Below the transition temperature for the emergence of stable dimeric gas, from Eq. (15), one has

$$\alpha\left(T\right) = 1 - \frac{3N_D\left(T=0\right)}{2N} \left[1 + \left(\frac{\pi^2 \xi_0}{3} - \frac{1}{\xi_0}\right)\xi - \frac{\pi^2 \xi^2}{3}\right]^3. \tag{28}$$

To compare with the experimental result in [12], from Eq. (27), one gets

$$\ln\left(\frac{E}{E_0} - 1\right) = \ln\left[\frac{2\pi^2}{3}\left(\frac{T}{T_F}\right)_{fit}^2\right] + \ln\alpha. \tag{29}$$

Without the factor $\ln\alpha$, the above equation gives the result for strongly interacting two-component Fermi gases without the dimeric gas. Thus, the factor $\ln\alpha$ on the right hand side of the above equation will give us clearly the role of the dimeric gas in the thermodynamic properties of the system. Below the transition temperature (i.e. in the presence of the dimeric gas), α is smaller than 1 and thus $\ln\alpha$ is negative. In this case, compared with the theoretical model without dimeric gas, in the relation between $\ln\left(E/E_0 - 1\right)$ and $\ln\left(T/T_F\right)_{fit}$, the presence of the dimeric gas will decrease significantly the value of $\ln\left(E/E_0 - 1\right)$. In the experimental measurement of the condensate fraction on resonance in [7], $2N_D\left(T=0\right)/N$ can be estimated to be 0.8 in the limit of zero temperature. In this case, the factor α is estimated to be 0.4. Thus in the limit of zero temperature, one has $\ln\alpha = -0.92$ which means a significant decreasing of $\ln\left(E/E_0 - 1\right)$ when the presence of dimeric gas is considered. In fact, this effect has been observed in the experiment [12] which shows clearly the phase transition for the emergence of the dimeric gas through the investigation of the relation between $\ln\left(E/E_0 - 1\right)$ and $\ln\left(T/T_F\right)_{fit}$.

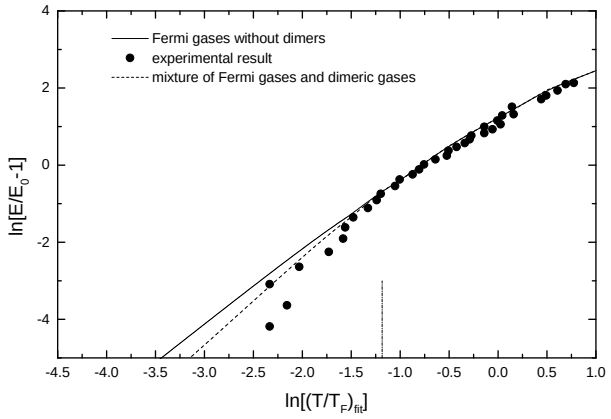

FIG. 1: Shown in figure 1 is the theoretical and experimental results of the relation between $\ln\left(E/E_0 - 1\right)$ and $\ln\left(T/T_F\right)_{fit}$. The circles show the experimental result by J. Kinast *et al* [12], while the solid line is the theoretical predication of the strongly interacting two-component Fermi gases without the consideration of the dimeric gas. It is clearly shown that there is a significant difference between the solid line and experimental data at low temperature which means that there is a phase transition for the strongly interacting Fermi gases. The dashed line is the theoretical result based on the consideration that there is a mixture of the Fermi gases and dimeric gas below a transition temperature. We see that this theoretical model agree well with the experimental result. The dash-dotted line shows the transition temperature $T_{tran} = 0.31\sqrt{1+\beta_F} T_F$ based on the consideration of the thermal excitation of the dimers, which is in agreement with the experimental result $0.33\sqrt{1+\beta_F} T_F$.

We now give a comparison of our theoretical result with the recent experimental result in [12]. In this experiment by Kinast *et al*, an equal mixture of the degenerate Fermi gases is prepared for two lowest spin states of ^6Li atoms, and the magnetic field is tuned to be 840 G which is just above the Feshbach resonant magnetic field. For the magnetic field of 840 G, the absolute value of the scattering length is much larger than the average distance between particles so that our theoretical model can be used to analysis the experimental result. From Eqs. (18), (22) and (26), the dashed line shown in Fig.1 is the numerical result of the relation between $\ln\left(E/E_0 - 1\right)$ and $\ln\left(T/T_F\right)_{fit}$ by using the transition temperature $T_{tran} = 0.31\sqrt{1+\beta_F} T_F$ obtained in this paper. The circles show the experimental result in [12], while the solid line shows the theoretical result based on the model of two-component Fermi gases without dimeric gas. We see that below the transition temperature, the theoretical model based the mixture of two-component Fermi gases and dimeric gas agrees with the experimental result.

V. SUMMARY AND DISCUSSION

In summary, we investigate the thermodynamic properties of the strongly interacting two-component Fermi gases by considering specially the role of the dimeric gas. Due to the divergent scattering length between dimers, we assume that the dimers satisfy Fermi statistics and there is a university behavior for the dimeric gas. It is found that this simple theoretical model agrees with the recent experiments about the condensate fraction, collective excitations, transition temperature and energy of the ultracold gases near the magnetic-field Feshbach resonance. The model proposed here is a phenomenological theory just like the effective force is introduced to describe the atomic nucleus which is a complex, many-body system bound by the strong interaction. Nevertheless, the agreement with the experimental results gives us a strong support for the validity of our theoretical model although the microscopic origin of our theory is still remained to be revealed. In future work, we will try to find the microscopic mechanism of the theoretical model proposed here.

Note added.−In preparing this manuscript, we noticed a recent work by Q. Chen *et al* [40] where a theoretical model is proposed based on the mixture of fermionic atoms and hybridized bosons. There is an astonishing consistency between the theoretical result obtained by Q. Chen *et al* [40] and our theoretical result although the dimer is assumed by us to satisfy Fermi statistics due to its divergent scattering length.

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